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WEIGHTING FUNCTION AND TRANSIENT THERMAL RESPONSE OF BUILDINGS
PART II—COMPOSITE STRUCTURE

Z. U. A. WARSI and N. K. D. CHOUDHURY
Central Building Research Institute, Roorkee, India

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Abstract—In this paper the concept and use of weighting functions are extended to enclosures having walls and roofs of composite construction. The theory developed has been used to compute the weighting functions and thermal response in a few typical cases, such as cavity walls and insulated walls with a cladding of external or internal insulation.

NOMENCLATURE

A_n, constants defined in the text, $n = 1, 2, \ldots$;
$A_F(t)$, temperature response of the indoor air to unit step change of outside temperature [degC];
$F(t)$, function of time defining an arbitrary outdoor temperature variation;
C_a, thermal capacity of the air expressed in terms of per unit area of the exposed walls [kcal/m² degC];
C_i, thermal capacity of the internal mass expressed in terms of per unit area of the exposed walls [kcal/m² degC];
$\theta_1(x, t)$, temperature at position x in the external layer at time t [°C];
$\theta_2(x, t)$, temperature at position x in the second layer at time t [°C];
$\theta_3(x, t)$, temperature at position x in the third layer at time t [°C];
$\theta_i(t)$, temperature of the inside air [°C];
$\theta_{is}(t)$, temperature of internal mass [°C];
$1/R_i$, inside wall surface heat-transfer coefficient [kcal/m² h degC];
$1/R_0$, outside wall surface heat-transfer coefficient [kcal/m² h degC];
$1/R_s$, heat-transfer coefficient at the surface of internal mass [kcal/m² h degC];
k_1, k_2, k_3, thermal diffusivities of the respective layers [m²/h];
K_1, K_2, K_3, thermal conductivities of the respective layers [kcal/m h degC];
$l_1, l_2 - l_1, l_3 - l_2$, thicknesses of the consecutive layers [m];
σ, conductance of the air cavity [kcal/m² h degC];
t, time [h];
T, thermal time constant [h];
x, position in the layer [m];
ξ, variable of integration;
m, number of air changes per hour [1/h];
p, Laplace transform parameter;
$\phi_F(t)$, weighting function [1/h];
$\psi(p)$, function of Laplace transform parameter p;
β_n, roots of the transcendental equation occurring in the text, $n = 1, 2, \ldots$;
$\chi(\beta_n)$, function of the roots β_n;
x_1, x_2, y_1, y_2, constants defined in the text;

$$C_1 = \sqrt{\left(\frac{k_1}{k_2}\right)}\frac{K_2}{K_1};$$

$$C_2 = \sqrt{\left(\frac{k_2}{k_3}\right)}\frac{K_3}{K_2};$$

$$L_2 = l_2 - l_1;$$

$$L_3 = l_3 - l_2.$$

INTRODUCTION

IN THE previous paper [1], the theory and concept of weighting functions was developed for

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enclosures with walls and roofs of homogeneous construction. To complete the investigation on thermal response by means of weighting functions, the theory will be extended to composite structures. The situation considered is of practical importance as most of the enclosures are bounded by walls having more than one layer of different material composition. In this paper, for the sake of mathematical brevity, we again refer to the paper of Pratt and Ball as [2].

RESPONSE TO A UNIT STEP PULSE

Let us consider a structure having composite walls and roof of three layers, the thicknesses of which are l_1, $l_2 - l_1$, $l_3 - l_2$, and conductivities K_1, K_2, K_3 respectively (Fig. 1). We

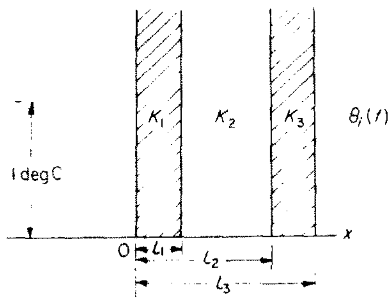

FIG. 1 Diagrammetric representation of a composite wall.

assume that there is perfect thermal contact at the interfaces. The system is considered at a temperature 0°C at $t = 0$ when the outside temperature instantaneously rises to 1°C. The equations governing the flow of heat in an enclosure following a unit step function of temperature are:

$$k_1 \frac{\partial^2 \theta_1}{\partial x^2} = \frac{\partial \theta_1}{\partial t}, \qquad 0 < x < l_1, \quad t > 0 \tag{1}$$

$$k_2 \frac{\partial^2 \theta_2}{\partial x^2} = \frac{\partial \theta_2}{\partial t}, \qquad l_1 < x < l_2, \quad t > 0 \tag{2}$$

$$k_3 \frac{\partial^2 \theta_3}{\partial x^2} = \frac{\partial \theta_3}{\partial t}, \qquad l_2 < x < l_3, \quad t > 0 \tag{3}$$

$$-K_1 \frac{\partial \theta_1}{\partial x} = \frac{1}{R_0}(1 - \theta_1), \; x = 0, \qquad t > 0 \tag{4}$$

$$\theta_1 = \theta_2, \qquad x = l_1, \quad t > 0 \tag{5}$$

$$K_1 \frac{\partial \theta_1}{\partial x} = K_2 \frac{\partial \theta_2}{\partial x}, \qquad x = l_1, \quad t > 0 \tag{6}$$

$$\theta_2 = \theta_3, \qquad x = l_2, \quad t > 0 \tag{7}$$

$$K_2 \frac{\partial \theta_2}{\partial x} = K_3 \frac{\partial \theta_3}{\partial x}, \qquad x = l_2, \quad t > 0 \tag{8}$$

$$-K_3 \frac{\partial \theta_3}{\partial x} = \frac{1}{R_i}(\theta_3 - \theta_i), \; x = l_3, \qquad t > 0 \tag{9}$$

$$C_i \frac{d\theta_{is}}{dt} = \frac{1}{R_s}(\theta_i - \theta_{is}), \qquad t > 0 \tag{10}$$

$$\frac{1}{R_i}(\theta_3 - \theta_i) + m\,C_a(1 - \theta_i) = \frac{1}{R_s}(\theta_i - \theta_{is}), \qquad x = l_3, \quad t > 0 \tag{11}$$

SOLUTION OF THE EQUATIONS

The method of solution adopted here is the same as in Part I. Taking Laplace-transform of the equations (1)–(11), we have after elimination and simplification

$$\bar{\theta}_i = \frac{(1 + p\,C_i\,R_s)\,\{4\,\alpha_3\,K_3\,R_i - m\,C_a\,R_i\,[y_2\,(1 - \alpha_3\,K_3\,R_i)\exp(-\alpha_3\,L_3) - y_1\,(1 + \alpha_3\,K_3\,R_i)\exp(\alpha_3\,L_3)]\}}{p\,(1 + p\,C_i\,R_s)\,[y_2\exp(-\alpha_3\,L_3) - y_1\exp(\alpha_3\,L_3)] - p\,\psi(p)\,[y_2\,(1 - \alpha_3\,K_3\,R_i)\exp(-\alpha_3\,L_3) - y_1\,(1 + \alpha_3\,K_3\,R_i)\exp(\alpha_3\,L_3)]} \tag{12}$$

where

$$\left.\begin{aligned}
y_1 &= x_2 (1 - C_2) \exp(-\alpha_2 L_2) + x_1 (1 + C_2) \exp(\alpha_2 L_2) \\
y_2 &= x_2 (1 + C_2) \exp(-\alpha_2 L_2) + x_1 (1 - C_2) \exp(\alpha_2 L_2) \\
x_1 &= (\alpha_1 K_1 R_0 + C_1) \sinh \alpha_1 l_1 + (1 + \alpha_2 K_2 R_0) \cosh \alpha_1 l_1 \\
x_2 &= (\alpha_1 K_1 R_0 - C_1) \sinh \alpha_1 l_1 + (1 - \alpha_2 K_2 R_0) \cosh \alpha_1 l_1 \\
C_1 = \sqrt{\left(\frac{k_1}{k_2}\right)} \frac{K_2}{K_1}, \quad C_2 &= \sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2}, \quad \alpha_1 = \sqrt{\left(\frac{p}{k_1}\right)}, \quad \alpha_2 = \sqrt{\left(\frac{p}{k_2}\right)}, \quad \alpha_3 = \sqrt{\left(\frac{p}{k_3}\right)} \\
L_2 &= l_2 - l_1, \quad L_3 = l_3 - l_2
\end{aligned}\right\} \tag{13}$$

and

$$\psi(p) = (2 + m\, C_a R_i)(1 + p\, C_i R_s) - 1 \tag{14}$$

$$\bar{\theta}_i = \int_0^\infty \mathrm{e}^{-pt} \theta_i(t)\, \mathrm{d}t \tag{15}$$

After inversion of (12) by the method detailed in [2], we get

$$\theta_i(t) = 1 - \sum_{n=1}^{\infty} \frac{A_n \exp(-\beta_n^2 t)}{\beta_n^2\, \chi(\beta_n)} \tag{16}$$

where

$$\left.\begin{aligned}
A_n = (1 - C_i R_s \beta_n^2) \Bigg(& \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} + m\, C_a R_i \Bigg\{ \left(\cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \times \\
& \left[\sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} + \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \right. \\
& \left. \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{\beta_n K_3^2 R_i \sqrt{(R\sqrt{(k_2)}_i)}}{k_3 K_2} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} \right] - \\
& \left[\sqrt{\left(\frac{k_1}{k_2}\right)} \frac{K_2}{K_1} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_2 R_0}{\sqrt{(k_2)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} \right] \left[\sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} - \right. \\
& \left. \sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} \right] - \\
& \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} \left[\sqrt{\left(\frac{k_1}{k_2}\right)} \frac{K_2}{K_1} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_2 R_0}{\sqrt{(k_2)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} \right] \times \\
& \left[\sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} + \sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} \right] \Bigg\} \Bigg)
\end{aligned}\right\} \tag{17}$$

and

$$
\begin{aligned}
\chi(\beta_n) = {} & [(1 - C_i R_s \beta_n^2)(2 + m C_a R_i) + C_i R_s \beta_n^2 - 2] \left\{ \left(\frac{l_1}{2 \beta_n \sqrt{(k_1)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} + \right.\right. \\
& \left. \frac{K_1 R_0}{2 \beta_n \sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0 l_1}{2 k_1} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \left[\cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \right. \\
& \left. \sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} \right] + \left(\cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \times \\
& \left(\frac{L_2}{2 \beta_n \sqrt{(k_2)}} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{L_3}{2 \beta_n \sqrt{(k_3)}} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} - \right. \\
& \left. \frac{L_2 K_3}{2 \beta_n K_2 \sqrt{(k_3)}} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{L_3 K_3 \sqrt{(k_2)}}{2 \beta_n K_2 k_3} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_2}{\sqrt{(k_3)}} \right) + \\
& \left(\frac{l_1 K_2}{2 \beta_n K_1 \sqrt{(k_2)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_2 R_0}{2 \beta_n \sqrt{(k_2)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{K_2 R_0 l_1}{2 \sqrt{(k_1 k_2)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \times \\
& \left[\sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} - \sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} \right] + \\
& \left[\sqrt{\left(\frac{k_1}{k_2}\right)} \frac{K_2}{K_1} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_2 R_0}{\sqrt{(k_2)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} \right] \left(\frac{L_2}{2 \beta_n \sqrt{(k_2)}} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \right. \\
& \frac{L_3}{2 \beta_n \sqrt{(k_3)}} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3 \sqrt{(k_2)} L_3}{2 \beta_n k_3 K_2} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \\
& \left.\left. \frac{K_3 L_2}{2 \beta_n \sqrt{(k_3)} K_2} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} \right) \right\} + \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} \times \\
& [(2 + m C_a R_i)(1 - C_i R_0 \beta_n^2) - 1] \left\{ \left(\frac{l_1}{2 \beta_n \sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0}{2 \beta_n \sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \right.\right. \\
& \left. \frac{K_1 R_0 l_1}{2 k_1} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \left[\cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} - \sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} \right] + \\
& \left(\cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \left(\frac{L_2}{2 \beta_n \sqrt{(k_1)}} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} + \right. \\
& \frac{L_3}{2 \beta_n \sqrt{(k_3)}} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3 L_2}{2 \beta_n \sqrt{(k_3)} K_2} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \\
& \left. \frac{K_3 \sqrt{(k_2)} L_3}{2 \beta_n k_3 K_2} \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} \right) + \left(\frac{l_1 K_2}{2 \beta_n K_1 \sqrt{(k_2)}} \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_2 R_0}{2 \beta_n \sqrt{(k_2)}} \right. \\
& \left. \cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{K_2 R_0 l_1}{2 \sqrt{(k_1 k_2)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} \right) \left[\sqrt{\left(\frac{k_2}{k_3}\right)} \frac{K_3}{K_2} \cos \frac{\beta_n L_2}{\sqrt{(k_2)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}} + \sin \frac{\beta_n L_2}{\sqrt{(k_2)}} \times \right.
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\cos\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big] + \Big[\sqrt{\Big(\frac{k_1}{k_2}\Big)}\,\frac{K_2}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_2 R_0}{\sqrt{(k_2)}}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big]\Big(\frac{K_3 L_3 \sqrt{(k_2)}}{2\,\beta_n K_2 k_3}\\
&\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{K_3 L_2}{2\,\beta_n\sqrt{(k_3)}\,K_2}\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{L_2}{2\,\beta_n\sqrt{(k_2)}}\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\\
&\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{L_3}{2\,\beta_n\sqrt{(k_3)}}\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big)\Big\} + C_i R_s (1 + m\,C_a R_i)\\
&\Big\{\sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\Big(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0 \beta_n}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big)\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} +\\
&\Big(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big)\times\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} - \Big[\sqrt{\Big(\frac{k_1}{k_2}\Big)}\,\frac{K_2}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\\
&+\frac{\beta_n K_2 R_0}{\sqrt{(k_2)}}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big]\times\Big[\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} - \sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big]\Big\} +\\
&\frac{K_3 R_i}{\sqrt{(k_3)}}\Big\{(2 + m\,C_a R_i)\Big[C_i R_s \beta_n - \frac{1}{2\,\beta_n}(1 - C_i R_s \beta_n^2)\Big] + \frac{1}{2\,\beta_n}\Big\}\ \Big\{\Big(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} -\\
&\frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big)\Big[\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big] -\\
&\Big[\sqrt{\Big(\frac{k_1}{k_2}\Big)}\,\frac{K_2}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_2 R_0}{\sqrt{(k_2)}}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big]\Big[\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} +\\
&\sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big]\Big\}
\end{aligned}
\tag{18}
$$

and β_n are the roots of the equation

$$
\begin{aligned}
&[2 - C_i R_s \beta_n^2 - (1 - C_i R_s \beta_n^2)(2 + m\,C_a R_i)]\Big\{\Big(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{K_1 R_0 \beta_n}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big)\times\\
&\Big[\sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} + \cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big] + \Big[\sqrt{\Big(\frac{k_1}{k_2}\Big)}\,\frac{K_2}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} +\\
&\frac{\beta_n K_2 R_0}{\sqrt{(k_2)}}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big]\ \Big[\sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big]\Big\} +\\
&\frac{\beta_n K_3 R_i}{\sqrt{(k_3)}}[1 - (1 - C_i R_s \beta_n^2)(2 + m\,C_a R_i)]\times\Big\{\Big(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big)\\
&\Big[\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big] - \Big[\sqrt{\Big(\frac{k_1}{k_2}\Big)}\,\frac{K_2}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} +\\
&\frac{\beta_n K_2 R_0}{\sqrt{(k_2)}}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\Big]\ \Big[\sin\frac{\beta_n L_2}{\sqrt{(k_2)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} + \sqrt{\Big(\frac{k_2}{k_3}\Big)}\,\frac{K_3}{K_2}\cos\frac{\beta_n L_2}{\sqrt{(k_2)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Big]\Big\} = 0.
\end{aligned}
\tag{19}
$$

In particular the thermal response of an enclosure bounded by walls of two layers can easily be deduced by putting l_1 or $l_3 - l_2 = L_3$ equal to zero.

WEIGHTING FUNCTION

The theory and concept of weighting function of enclosures have been described in Part I of this paper [1]. The same concept can now be extended to enclosures having composite construction. The weighting function for this case is obtained by differentiating equation (16) with respect to t, i.e.

$$\phi_F(t) = \sum_{n=1}^{\infty} \frac{A_n \exp(-\beta_n^2 t)}{\chi(\beta_n)}$$

where A_n, $\chi(\beta_n)$ and β_n have been defined in equations (17), (18) and (19).

Having calculated the weighting function we can use it for the calculation of thermal response of the enclosure subjected to arbitrary outdoor temperature change $F(t)$ with the help of the convolution integral

$$\theta_i(t) = \int_0^t F(t-\xi)\, \phi_F(\xi)\, \mathrm{d}\xi.$$

CAVITY WALL

We now consider an enclosure bounded by cavity walls. The cavity is of uniform width and encloses air which is assumed to be unventilated. Since the thermal capacity of air is zero, we can reduce the general solution for a three layered structure as obtained in (16) to that for cavity walls simply by taking the limit as $k_2 \to \infty$. This reduction introduces a parameter $\sigma = K_2/L_2$, which can easily be identified as the conductance of the cavity. The heat transfer across the cavity has therefore been taken into account through a finite conductance σ of the air cavity. It has been shown [3] that when the width of the air cavity exceeds 2 cm, its effective conductance remains unchanged and it includes the combined influence of convective, radiative and conductive heat transfer. With this consideration, σ is invariant; hence the boundary conditions (6) and (8) reduce to

$$K_1 \frac{\partial \theta_1}{\partial x} = K_3 \frac{\partial \theta_3}{\partial x} \quad \text{at } x = l_1, \quad t > 0$$

which is the same as equations (1.3) and (1.5) in Appendix of [2].

Let the two embedding layers be of conductivities K_1 and K_3, diffusivities k_1 and k_3 and thicknesses l_1 and $l_3 - l_2 = L_3$, respectively. The thickness of the air cavity is then L_2 of conductivity K_2. The thermal response $A_F(t)$ now follows from (16), which reduces to

$$A_F(t) = 1 - \sum_{n=1}^{\infty} \frac{A_n \exp(-\beta_n^2 t)}{\beta_n^2 \chi(\beta_n)} \tag{20}$$

where now

$$\left.\begin{aligned}
A_n = (1 - C_i R_s \beta_n^2)\Bigg(&\frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} + m C_a R_i \Bigg\{\left(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}} \sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\right) \times \\
&\left(\frac{K_3 \beta_n}{\sigma\sqrt{(k_3)}} \cos\frac{\beta_n L_3}{\sqrt{(k_3)}} + \sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} \cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{\beta_n^2 R_i K_3^2}{\sigma k_3} \sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\right) + \\
&\left[\sqrt{\left(\frac{k_1}{k_3}\right)} \frac{K_3}{K_1} \sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_3 R_0}{\sqrt{(k_3)}} \cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\right] \cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \\
&\frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} \left[\sqrt{\left(\frac{k_1}{k_3}\right)} \frac{K_3}{K_1} \sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{\beta_n K_3 R_0}{\sqrt{(k_3)}} \cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\right] \sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\Bigg\}\Bigg)
\end{aligned}\right\} \tag{21}$$

and

$$
\begin{aligned}
\chi(\beta_n) = {} & [(1 - C_i R_s \beta_n^2)(2 + m C_a R_i) + C_i R_s \beta_n^2 - [2] \bigg[\bigg(\frac{l_1}{2\beta_n\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \\
& \frac{K_1 R_0}{2\beta_n\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0 l_1}{2k_1}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\bigg(\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{\beta_n K_3}{\sigma\sqrt{(k_3)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg) - \\
& \bigg(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\times\bigg(\frac{L_3}{2\beta_n\sqrt{(k_3)}}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3}{2\beta_n\sigma\sqrt{(k_3)}} \\
& \cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{L_3 K_3}{2\sigma k_3}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg) - \frac{K_3}{\sqrt{(k_3)}}\bigg(\frac{l_1}{2\beta_n K_1}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{R_0}{2\beta_n}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} \\
& - \frac{R_0 l_1}{2\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\times\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3 L_3}{2\beta_n\sqrt{(k_3)}}\bigg(\frac{\sqrt{(k_1)}}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \beta_n R_0 \\
& \cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg] + \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}}[(2 + m C_a R_i)(1 - C_i R_s \beta_n^2) - 1] \\
& \bigg[\bigg(\frac{l_1}{2\beta_n\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0}{2\beta_n\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0 l_1}{2\sqrt{(k_1)}}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg) \\
& \bigg(\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{K_3\beta_n}{\sigma\sqrt{(k_3)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg) + \bigg(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg) \\
& \bigg(\frac{L_3}{2\beta_n\sqrt{(k_3)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3}{2\sigma\beta_n\sqrt{(k_3)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3 L_3}{2\sigma k_3}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg) + \frac{K_3}{\sqrt{(k_3)}} \\
& \bigg(\frac{l_1}{2\beta_n K_1}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{R_0}{2\beta_n}\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{R_0 l_1}{2\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \\
& \bigg(\frac{\sqrt{(k_1)}}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \beta_n R_0\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\frac{K_3 L_3}{2\beta_n k_3}\cos\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg] + C_i R_s (1 + m C_a R_i) \\
& \bigg[\frac{K_3\beta_n}{\sigma\sqrt{(k_3)}}\bigg(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} + \frac{K_1 R_0 \beta_n}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} + \bigg(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \\
& \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\sin\frac{\beta_n L_3}{\sqrt{(k_3)}} + \frac{K_3}{\sqrt{(k_3)}}\bigg(\frac{\sqrt{(k_1)}}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \beta_n R_0\cos\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg) \\
& \cos\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg] + \frac{K_3 R_i}{\sqrt{(k_3)}}\bigg\{(2 + m C_a R_i)\bigg[C_i R_s \beta_n - \frac{1}{2\beta_n}(1 - C_i R_s \beta_n^2)\bigg] + \frac{1}{2\beta_n}\bigg\} \\
& \bigg[\bigg(\cos\frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\bigg(\cos\frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{\beta_n K_3}{\sigma\sqrt{(k_3)}}\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg) - \\
& \frac{K_3}{\sqrt{(k_3)}}\bigg(\frac{\sqrt{(k_1)}}{K_1}\sin\frac{\beta_n l_1}{\sqrt{(k_1)}} + \beta_n R_0\sin\frac{\beta_n l_1}{\sqrt{(k_1)}}\bigg)\sin\frac{\beta_n L_3}{\sqrt{(k_3)}}\bigg]
\end{aligned}
\tag{22}
$$

and β_n are now given by

$$\left.\begin{array}{l}[2 - C_i R_s \beta_n^2 - (1 - C_i R_s \beta_n^2)(2 + m C_a R_i)] \left[\left(\cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{K_1 R_0 \beta_n}{\sqrt{(k_1)}} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}}\right)\right. \\ \left(\frac{\beta_n K_3}{\sigma \sqrt{(k_3)}} \cos \frac{\beta_n L_3}{\sqrt{(k_3)}} + \sin \frac{\beta_n L_3}{\sqrt{(k_3)}}\right) + \frac{K_3}{\sqrt{(k_3)}} \left(\frac{\sqrt{(k_1)}}{K_1} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \beta_n R_0 \cos \frac{\beta_n l_1}{\sqrt{(k_1)}}\right) \\ \left.\cos \frac{\beta_n L_3}{\sqrt{(k_3)}}\right] + \frac{\beta_n K_3 R_i}{\sqrt{(k_3)}} [1 - (1 - C_i R_s \beta_n^2)(2 + m C_a R_i)] \left[\left(\cos \frac{\beta_n l_1}{\sqrt{(k_1)}} - \frac{\beta_n K_1 R_0}{\sqrt{(k_1)}}\right.\right. \\ \left.\sin \frac{\beta_n l_1}{\sqrt{(k_1)}}\right) \left(\cos \frac{\beta_n L_3}{\sqrt{(k_3)}} - \frac{\beta_n K_3}{\sigma \sqrt{(k_3)}} \sin \frac{\beta_n L_3}{\sqrt{(k_3)}}\right) - \frac{K_3}{\sqrt{(k_3)}} \left(\frac{\sqrt{(k_1)}}{K_1} \sin \frac{\beta_n l_1}{\sqrt{(k_1)}} + \right. \\ \left.\left.\beta_n R_0 \cos \frac{\beta_n l_1}{\sqrt{(k_1)}}\right) \sin \frac{\beta_n L_3}{\sqrt{(k_3)}}\right] = 0.\end{array}\right\} \quad (23)$$

THERMAL TIME CONSTANT

The thermal time constant T of an enclosure is defined as the time in which the internal air temperature rises to 63 per cent of its final steady value. The time constant in a thermal circuit is analogous to that in an electrical circuit having capacitance and resistance. Now considering the expression for the air temperature given in (16) and keeping only the first term in the series we have

$$\theta_i(t) \simeq 1 - \frac{A_1 \exp(-\beta_1^2 t)}{\beta_1^2 \chi(\beta_1)}. \quad (24)$$

This form of $\theta_i(t)$ may be considered as a sufficiently good approximation since the other terms approach zero more rapidly in comparison with that of the first term for $t \geqslant T$. Therefore the time in which air temperature $\theta_i(t)$ reaches 0·63 is given by

$$\frac{A_1 \exp(-\beta_1^2 T)}{\beta_1^2 \chi(\beta_1)} = \frac{1}{e}$$

from which

$$T = \frac{1}{\beta_1^2} \log_e \left[\frac{e A_1}{\beta_1^2 \chi(\beta_1)}\right] \quad (25)$$

An approximate value of the thermal time constant can be taken as $1/\beta_1^2$ since $A_1/\beta_1^2 \chi(\beta_1)$ is mostly of the order of unity. The values of the thermal time constants for the particular cases in which numerical computations have been carried out are given in Table 1.

THERMAL RESPONSE

In the Field Project of this Institution, a few full sized test houses of homogeneous and composite constructions have been built up [1]. Insulating materials composed of foam plastic, reed board, light weight aggregate, etc., have been applied both as external and internal cladding on a core of thin masonry.

With a view to study the relative transient response of composite and homogeneous constructions a few typical cases of insulated and uninsulated masonry as given in Table 1, are chosen. These constructions have been used in the test houses and tested for their response under unsteady conditions of heat flow. The north wall is common with the anteroom and hence it is treated as an internal mass (C_i = 8 kcal/m^2 degC). In order to compare the performance under an idealized conditions of all exposed walls, enclosures having no internal mass ($C_i = 0$) are also postulated.

The thermal response to unit step function of a few composite constructions without internal mass are shown in Fig. 2. The response of 11·2 cm (4·5 in) brick wall is very sharp attaining 63 per cent of the outdoor temperature in only 5 h. The time constants of the constructions are given in Table 1. Compared to a 22·4 cm (9·0 in) usual brick wall, a cavity wall 27·4 cm (11 in) having 5 cm (2 in) air cavity sandwiched between two 11·2 cm thick brick layers is seen to function

Table 1. Data used for computation of weighting functions

Thermal properties of materials	Wall construction	Vol/Exposed wall area	C_a (kcal/ m² °C)	C_i (kcal/ m² degC)	Roots and constants β_1	β_2	$A_1/\chi(\beta_1)$	$A_2/\chi(\beta_2)$
1. Brick $K = 0{\cdot}87$ kcal/m h degC $k = 2{\cdot}4 \times 10^{-3}$ m²/h	Brick 11·2 cm	0·5	0·15	0	0·49	0·94	0·291	−0·186
	ditto	0·6	0·18	10	0·355	0·66	0·134	−0·026
2. Foam plastic $K = 0{\cdot}027$ kcal/m h degC	Brick 11·2 cm + 2·54 cm foam plastic inside	0·5	0·15	0	0·5	1·10	0·336	−0·36
$k = 3{\cdot}6 \times 10^{-3}$ m²/h	ditto	0·6	0·18	10	0·26	0·53	0·073	−0·027
3. Air cavity $\sigma = 5{\cdot}85$ kcal/m² h degC	Brick 22·4 cm	0·5	0·15	0	0·28	0·78	0·096	−0·123
	ditto	0·6	0·18	10	0·24	0·48	0·061	−0·019
$R_i = 1/6$ m²h degC/kcal	Cavity wall 27·4 cm (air cavity 5 cm)	0·5	0·15	0	0·25	0·85	0·070	−0·116
$R_0 = 1/20$ m²h degC/kcal	ditto	0·6	0·18	10	0·22	0·80	0·049	−0·038
	Brick 11·2 cm + 2·54 cm foam plastic outside	0·5	0·15	0	0·16	0·66	0·026	−0·039
	ditto	0·6	0·18	10	0·14	0·57	0·02	−0·016

Notes: The enclosure is considered unventilated, $m = 0$. Half of the north wall (22·4 cm brick) together with some commodities inside is considered as internal mass, $C_i = 8$ kcal/m² degC.

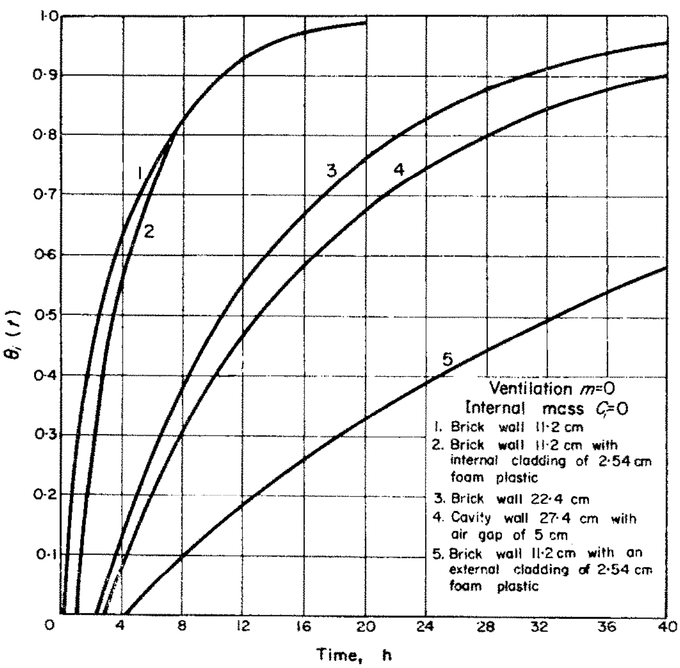

FIG. 2. Response to a unit step temperature of a room without internal mass.

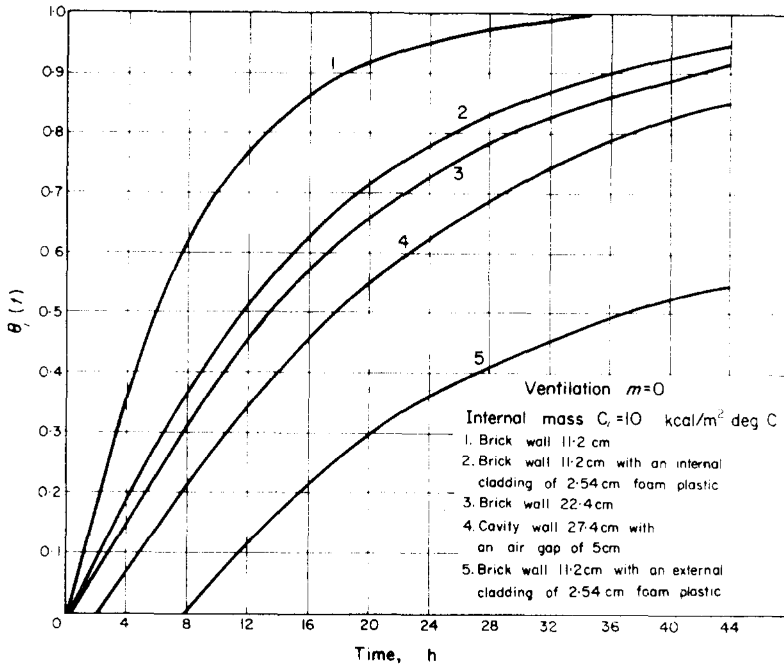

FIG. 3. Response to a unit step temperature of a room with internal mass.

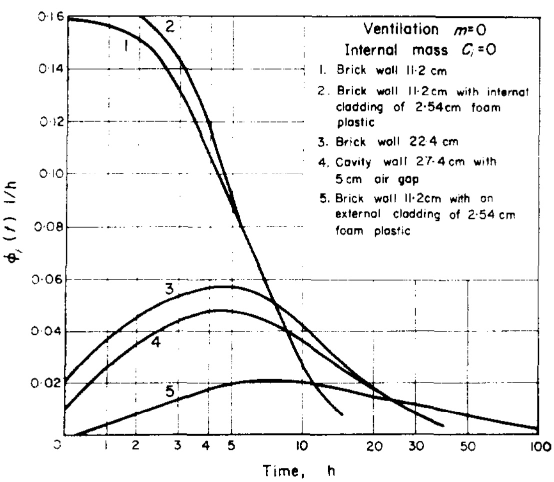

FIG. 4. Weighting functions of a room without internal mass.

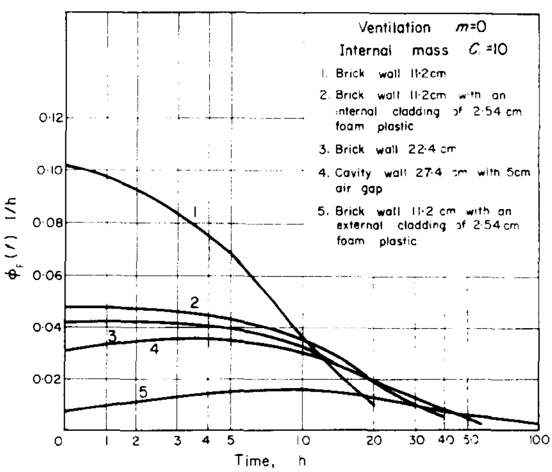

FIG. 5. Weighting functions of a room with internal mass.

Table 2. Thermal time constant, T in hours, of various structures

Structure	$m = 0, C_i = 0$	$m = 0, C_i = 10$
1. Brick 11·2 cm	4·0	8·38
2. Brick 11·2 cm + 2·54 cm foam plastic inside	4·8	16·2
3. Brick 22·4 cm	14·3	18·8
4. Cavity wall 27·4 cm (two layers of brick with 5 cm air cavity)	17·8	24·4
5. Brick 11·2 cm + 2·54 cm foam plastic outside	45·2	53·3

better in damping out the outside fluctuation during its transit through the wall. It is also observed that a foam plastic cladding, 2·54 cm (1·0 in) thick, on the exposed side of a 11·2 cm (4·5 in) brick wall is vastly superior to the same cladding on the inner side. This confirms the well-known fact that an insulating material is most effective in attenuating the influence of the outdoor temperature variation on the indoor climate when made the outside layer in a composite construction.

In the presence of internal mass the pattern is more or less the same (Fig. 3). The insulating efficiency of 2·54 cm (1 in) foam plastic on the inner side of a 11·2 cm (4·5 in), brick wall is improved and it becomes almost equivalent to a 22·4 cm (9·0 in) brick wall. The thermal time constants with internal mass are given in Table 2. The figures compare well with the thermal time constants T published earlier by this Institute [4], [5].

Figures 4 and 5 show the weighting functions for enclosures having composite construction with an without internal mass. The compared behaviour is as discussed above.

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Résumé—Dans cet article, le concept et l'utilisation de fonctions de pondération sont étendus aux enceintes ayant des murs et des plafonds de construction composite. La théorie développée a été employée pour calculer les fonctions de pondération et la réponse thermique dans quelques cas typiques, tels que des doubles parois et des murs isolés avec un revêtement d'isolement extérieur ou intérieur.

Zusammenfassung—Die Konzeption und die Anwendung der Gewichtsfunktionen wird auf Räume ausgedehnt, deren Wände und Dächer aus Schichten zusammengesetzt sind. Nach der entwickelten Theorie wurden die Gewichtsfunktionen und das thermische Verhalten für einige typische Fälle ausgerechnet, nämlich für unisolierte Wände eines Raumes und für solche, die mit einer äusseren oder inneren Isolierschicht bedeckt sind.

Аннотация—В данной статье понятие и применение весовых функций распространены на ограждения, стены и крыши которых имеют сложную конструкцию. Разработанная теория была использована для расчета весовых функций и тепловой реакции в нескольких типичных случаях, например, в случаях полых стен и изолированных стен при использовании внешней или внутрененней изоляции.